Route to low-scattering cylindrical cloaks with finite permittivity and permeability

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The method of coordinate transformation offers a way to realize perfect cloaks but provides less ability to characterize the performance of a multilayered cloak in practice. Here, we propose an analytical model to predict the performance of a multilayered cylindrical cloak, based on which, the cloak in practice can be optimized to diminish the intrinsic scatterings caused by discretization and simplification. Extremely low scattering or "quasiperfect invisibility" can be achieved with only a few layers of anisotropic metamaterials without following the transformation method. Meanwhile, the permittivity and permeability parameters of the layers are relatively small, which is a remarkable advantage of our approach.

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Various efforts have been made on the realization of invisibility.[1–5] Pendry *et al.* theoretically proposed the perfect invisibility cloak for electromagnetic waves,[1] utilizing anisotropic and inhomogeneous media to mimic the space squeezing. Later, the effectiveness of the transformation-based (TB) cloak was demonstrated by ray tracing,[6] full wave finite element simulations,[7,8] and analytical scattering models[9–12] as well. In practice, the difficulty in construction of a perfect invisibility cylindrical cloak is the requirement of continuous inhomogeneity and high anisotropy with extreme values in the parameters. Simplified parameters based on the coordinate transformation were then utilized to facilitate the physical realization,[7,13,14] in the expense of inherent scatterings.[15] Constraints on the bandwidth were studied as well.[16] The first sample of cylindrical cloak has been created using multilayered metamaterials.[13] Bi-layered isotropic media were also proposed for achieving the effective anisotropy[17] but a lot of thin layers are needed, which increases the construction complexity. Moreover, the transformation method provides less ability to predict the performance of a practical construction composed of discontinuous layers of homogeneous anisotropic metamaterials. Therefore, it is necessary to investigate a better way to design a practical cloak with good performance.

In this paper, in order to get the exact behavior of a multilayered cloak, the analytical model of a cylindrical cloak created with multilayered anisotropic materials is established based on the full wave scattering theory.[18,19] Through the optimization using the genetic algorithm method, we show that although only a few layers of anisotropic materials is used, a "quasiperfect" invisibility multilayered cloak with near zero scattering can still be achieved without following the design method of coordinate transformation, and the parameters obtained are relatively small and possible to be realized by metamaterials. The impedances between the adjoined layers do not really match each other but can produce near zero reflection, which can be treated as the counterpart in cylindrical geometry of the reflectionless one-dimensional multilayered slab. These results provide a second and better way of designing a multilayered cloak.

We use cylindrical cloak as an example. Without losing the generality, the case of a TE-polarized plane wave with unit magnitude normally incident onto an M-layer cylindrical cloak (from region 1 to M) is considered, as shown in Fig. 1. The TM case can be analyzed similarly. The radii of the boundaries of the cloak are denoted by R_m $(m=0,1,\ldots,M)$. The relative constitutive parameters in region m are assumed to be constants denoted by $\mu_{\rho m}$, $\mu_{\phi m}$, and ϵ_{zm} while the region $m=0$ is assumed to be free space and there is a thin perfect electric conductor (PEC) coating on the boundary between regions M and $M+1$. The electric fields E_{zm} in region m satisfy the following equation:

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\frac{\rho}{\mu_{\phi m}}\frac{\partial E_{zm}}{\partial\rho}\right)+\frac{1}{\rho^2}\frac{\partial}{\partial\phi}\left(\frac{1}{\mu_{\rho m}}\frac{\partial E_{zm}}{\partial\phi}\right)+k_0^2\epsilon_{zm}E_{zm}=0. \tag{1}$$

By applying the method of separation of variables, the general expression for the electric fields in region m can be expressed as

$$E_{zm}=\sum_{n=-\infty}^{\infty}a_{mn}[J_{\nu_{mn}}(k_m\rho)+\tilde{r}_{m(m+1)n}H_{\nu_{mn}}(k_m\rho)]\exp(in\phi), \tag{2}$$

where $\nu_{mn}=n\sqrt{\mu_{\phi m}/\mu_{\rho m}}$ and the wave number in region m is $k_m=k_0\sqrt{\epsilon_{zm}\mu_{\phi m}}$. Different from the isotropic layered case, here, ν_{mn} is a fraction. $J_{\nu_{mn}}$ and $H_{\nu_{mn}}$ represent the ν_{mn}th order Bessel functions of the first kind and the ν_{mn}th order Hankel functions of the first kind, respectively. a_{mn} is the unknown coefficient and $\tilde{r}_{m(m+1)n}$ is the scattering coefficient on the boundary R_m. When a standing-wave incident from

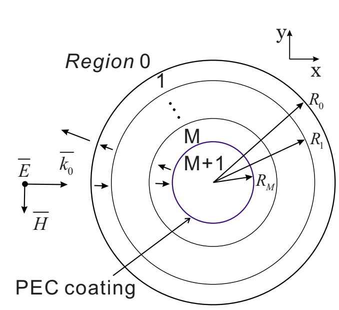

FIG. 1. (Color online) Configuration of a multilayered cylindrical cloak.

region m onto the boundary R_m, the direct reflection coefficient, which represents the ratio between the directly reflected wave and the incident wave, is $r_{m(m+1)n}=(j'j_1-\eta_m/\eta_{m+1}jj_1')/(-h'j_1+\eta_m/\eta_{m+1}hj_1')$, and the direct transmission coefficient, which represents the ratio between the directly transmitted wave in region $m+1$ and the incident wave in region m, is $t_{m(m+1)n}=-2i/(\pi k_m R_m)/(-h'j_1+\eta_m/\eta_{m+1}hj_1')$. Similarly, for an outgoing wave the direct reflection and transmission coefficients on R_m are $r_{(m+1)mn}=(h_1'h-\eta_{m+1}/\eta_m h_1 h')/(-j_1'h+\eta_{m+1}/\eta_m j_1 h')$ and $t_{(m+1)mn}=2i/(\pi k_{m+1}R_m)/(-j_1'h+\eta_{m+1}/\eta_m j_1 h')$. Here $j=J_{\upsilon_{mn}}(k_m R_m)$, $j'=J'_{\upsilon_{mn}}(k_m R_m)$, $j_1=J_{\upsilon_{(m+1)n}}(k_{m+1}R_m)$, $j_1'=J'_{\upsilon_{(m+1)n}}(k_{m+1}R_m)$, $h=H_{\upsilon_{mn}}(k_m R_m)$, $h'=H'_{\upsilon_{mn}}(k_m R_m)$, $h_1=H_{\upsilon_{(m+1)n}}(k_{m+1}R_m)$, $h_1'=H'_{\upsilon_{(m+1)n}}(k_{m+1}R_m)$, and $\eta_m=\sqrt{\mu_{\phi m}/\epsilon_{zm}}$ (Ref. 19). Therefore, the scattering coefficient in layer m ($m=0,1,\ldots,M-1$) can be written as[19]

$$\tilde{r}_{m(m+1)n}=r_{m(m+1)n}+\tilde{t}_{(m+1)mn}, \tag{3}$$

where $\tilde{t}_{(m+1)mn}=t_{m(m+1)n}t_{(m+1)mn}\tilde{r}_{(m+1)(m+2)n}/[1-r_{(m+1)mn}\tilde{r}_{(m+1)(m+2)n}]$ represents the wave coming out from R_m due to the multiple reflections and transmissions on the boundaries inside R_m. At R_M, $\tilde{r}_{M(M+1)n}=-J_{\nu_{Mn}}(k_M R_M)/H_{\nu_{Mn}}(k_M R_M)$, therefore all the $\tilde{r}_{m(m+1)n}$ can be derived using Eq. (3), and a_{mn} can also be derived by matching the boundary conditions. The coefficients of the scattering fields in region 0 are $b_{0n}=a_{0n}\tilde{r}_{01n}$. Based on the cylindrical scattering model, the far-field total scattering efficiency or the scattering cross section normalized by the geometrical cross section for the multilayered cylindrical cloak is obtained as

$$Q_{\text{sca}}=2/(k_0 R_M)\sum_{n=-\infty}^{\infty}|b_{0n}|^2. \tag{4}$$

When there is no PEC coating on the inner boundary, the above method is also applicable by simply setting the scattering coefficient in region $M+1$ to be 0 and using Eq. (3) for layer m ($m=0,1,\ldots,M$). However, the power will penetrate into region $M+1$ and objects put in region $M+1$ will affect the fields outside.

In practice, the ideal parameters obtained from the transformation method need to be discretized for realization which will destroy the perfect invisibility of the cloak. Using the proposed method, such effects of discretization and simplification of the transformation-based cloak can be quantitatively analyzed. For example, Schurig *et al.* proposed a ten-layer simplified cloak with $\mu_\phi=1$ for the experiment,[13] in which the copper cylinder core with radius 0.709λ is coated with a multilayered cloak with inner radius 0.768λ and outer radius 1.670λ. Utilizing the parameters including the losses in the metamaterials in Ref. 13 and assuming the core to be PEC for convenience, for the normal incidence of a TE-polarized plane wave, the near field distributions can be calculated using our method. Figure 2 shows the electric-field distributions of the above case. Our analytical model shows some qualitative agreement with the experimental field distributions shown in Ref. 13 where both reduced forward and backward scatterings can be observed.

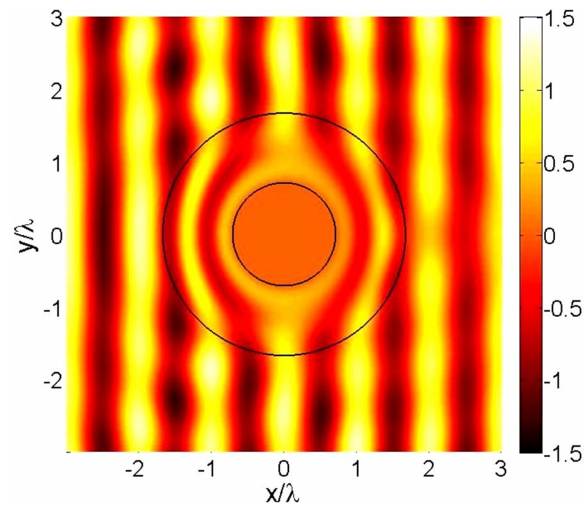

FIG. 2. (Color online) Electric-field distributions for a plane-wave incident from left to right onto the cloak proposed in Ref. 13.

A more interesting thing is that by using the genetic algorithms, a widely used optimization method in engineering and science that enables individuals of an optimization problem to evolve better solutions,[20] our proposed method can realize a general cloak without following the ideal transformation parameters but still have a quasiperfect performance. Meanwhile, the extreme values exist in the conventional transformation cloak can be avoided. When applying the genetic algorithms, the inner radius of the cloak is fixed and the chromosome is a string of 0s and 1s representing a set of constitutive parameters of each layer and the outer radius, $\{\epsilon_{z1},\mu_{\rho 1},\mu_{\phi 1},\ldots,\epsilon_{zM},\mu_{\rho M},\mu_{\phi M},R_0\}$. The value of each parameter is determined by a linear mapping from the integer denoted by the binary number to the search space. The thickness of each layer is set to be identical. The fitness of an individual is chosen to be $1/Q_{\text{sca}}$, where Q_{sca} is the total scattering of this individual shown by Eq. (4). Making use of the roulette wheel selection, in which individuals with larger fitness have greater chance of going forward to the next generation, setting the single point crossover probability and the mutation probability to be 0.6 and 0.05, respectively, and ensuring the fittest individual to propagate to the next generation. Thus, evolution is carried out and optimization is obtained finally.

Since in fabrications, it is much easier to create the cloak with fewer layers of matematerials, we consider a four-layer cloak as an example. The inner radius of the cloak is $R_4=0.709\lambda$. Considering the practical constraints on the constitutive parameters of the metamaterials, the search space of the relative parameters are between 0.01 and 8. And the outer radius R_0 is set between 0.8λ and 1.67λ. Using our proposed method, we design a four-layer optimized cloak with outer radius 0.894λ. The parameters are shown in Table I. Figure 3 shows the calculated electric field distribution when a TE-polarized plane wave normally incident from left to right onto the optimized cloak. It is shown that in the near region of the cloak, the electric fields stay almost unperturbed, thus a so-called quasiperfect cylindrical multilayered cloak is obtained. For the proposed parameters in practice, the values

TABLE I. The constitutive parameters for the optimized cloak and the TB full parameter cloak.

	Optimized cloak				TB full parameter cloak			
Layer	μ_ρ	μ_ϕ	ϵ_z	η/η_0	μ_ρ	μ_ϕ	ϵ_z	η/η_0
1	0.674	2.002	3.384	0.769	0.186	5.391	4.349	1.113
2	0.057	5.001	7.258	0.830	0.140	7.147	3.280	1.476
3	0.151	6.836	3.532	1.391	0.089	11.245	2.085	2.322
4	0.010	7.508	7.008	1.035	0.032	31.735	0.739	6.551

for μ_ρ are smaller than 1 and will be always dispersive. Since $\mu_{\rho 4}$ is very small, it will change quickly when frequency changes because of the dispersion. As a result, the performance will change quickly when the frequency changes. Therefore, similar to the transformation-based cloak, our proposed cloak works in a narrow band. The bandwidth is dependent on the dispersion of the material.

Figure 4 shows the far-field differential scattering efficiencies when a TE-polarized plane wave normally incident onto different types of cloaks. When there is only a bared PEC cylinder with radius 0.709λ, the differential scattering efficiency is shown by the solid line in Fig. 4, and the total scattering Q_{sca} is found to be 2.19. For Schurig's cloak (dashed), the forward and backward scatterings have been reduced by about 4.8 and 4.1 dB, respectively, and Q_{sca} is 0.49. When a TB full parameter cloak with the same dimensions as the quasiperfect one and with parameters shown in Table I is used (dash dot dotted), Q_{sca} is 0.11. When the optimized cloak is used (dotted), more than 20 dB improvement is achieved and Q_{sca} is greatly reduced to be 0.0039. For all the angles, the scatterings of the optimized cloak are so small that a quasiperfect invisibility cloak is achieved although only 4 layers are used. When the quasiperfect cloak does not have the PEC coating on the inner boundary and the inner region is free space (dash dotted), Q_{sca} is 0.44. Thus, from a different point of view, the PEC coating is the fifth layer of the proposed cloak and anything can be put into the inner region without affecting the fields outside. In this case, the proposed cloak has the same function as the TB cloak but with less layers and simpler parameters.

It is interesting to see from Table I that the optimized parameters are quite different from the parameters obtained by the coordinate transformation. Taking the ρ component as an example, as shown in Table I, the value of relative permeability in the most inner layer is optimized to be 0.01, instead of the value 0.032 as suggested by the transformation method. We also see that the parameters achieved here are relatively small and within the limit of metamaterials, which shows the possibility of realizing such a quasiperfect cloak. This is a very important contribution to the implementation of the cloak in practice. As we know, for an ideal cylindrical cloak, the ϕ component of the parameters will go infinitely near the inner boundary. Such kind of extreme value near the inner boundary is very difficult to realize. A truncation method[11] at the inner boundary can be used to avoid the extreme value of the inner boundary of the cloak. However, scattering will be aroused and it has been shown that the performance of such a cloak is sensitive to the perturbations on the inner boundary.[11] In order to get a performance as good as our proposed cloak, here Q_{sca} is 0.0039; only about $7\times 10^{-6}\lambda$ truncation is allowed on the inner boundary, which means huge values of permeability about $10^5\mu_0$ is needed in

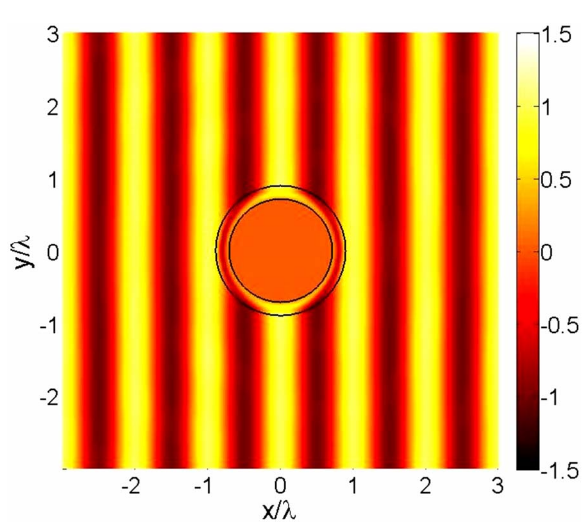

FIG. 3. (Color online) Electric-field distributions for a plane-wave incident from left to right onto the optimized quasiperfect cylindrical cloak.

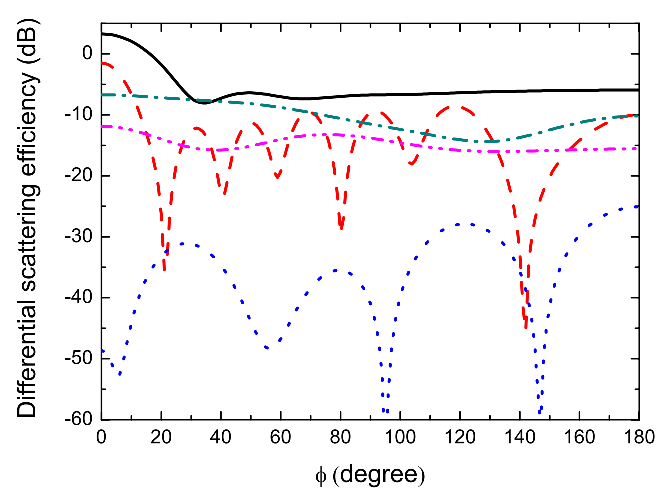

FIG. 4. (Color online) Comparison of the differential scattering efficiencies when a TE-polarized plane wave normally incident on to the PEC cylinder without cloak (solid), the PEC cylinder with the cloak proposed in Ref. 13 (dashed), a four-layer TB full parameter cloak (dash dot dotted), quasiperfect cloak (dotted), and the same quasiperfect cloak but without PEC coating (dash dotted).

the ϕ direction near the inner boundary. Such comparison reflects the advantage of our proposed cloak in practical implementation.

The reason that quasiperfect invisibility can still be achieved in a cloak with only a few layers can be physically explained as follows: the scattering of such kind of multilayered cloak is determined by the recurrence equation Eq. (3). When $m=0$, Eq. (3) indicates that the nth order scattering is the sum of the direct scattering at R_0 denoted by r_{01n} and the wave coming out from R_0 caused by the multiple reflections and transmissions on the inner boundaries, denoted by $\tilde{t}_{10n}$. In order to minimize the total scattering, the parameters of the cloak should be chosen so that r_{01n} and $\tilde{t}_{10n}$ cancel each other. And as denoted by Eq. (3), $\tilde{r}_{01n}$ are actually determined simultaneously by ν_{mn}, k_m, and η_m in all the layers. Thus, the match of impedances between the conjoined layers does not assure a small total scattering, while on the contrary, the mismatch of the impedances, as shown in Table I, can be utilized to form multiple reflections and transmissions among the inner layers, which eventually produce a transmission to the free space being able to destructively interfere with the direct reflection occurring at the outer boundary. In our proposed quasiperfect cloak, the zeroth direct reflection coefficient at R_0 is $r_{010}=-0.1135-0.3172i$, while the zeroth transmission coefficient from R_0 is $\tilde{t}_{010}=0.1130+0.2947i$; therefore, the zeroth scattering coefficient is $\tilde{r}_{010}=-0.0005-0.0225i$ which is very small due to the destructive interference. This is also similar to the one-dimensional multilayered case, where the impedances of each layer are not necessary to be matched in order to get zero reflection. The cloaks using the plasmonic shells also adopt the cancellation of scattering,[3,4] however, they are preferable for particles with small cross sections. Differently, the transformation-based cloak is applicable to larger objects and independent of the cloaked object, however, the performance of the practical transformation-based cloak is dependent on the discretization of the material parameters. Our proposed cloak is a type of cloak that falls in between but takes both the advantages of the above two methods, i.e., the transformation-based cloak and the scattering cancellation cloak using plasmonic shells. It uses a multilayered scheme and is applicable for larger objects. From Fig. 3, we can see that the phase distributions of our proposed cloak are very similar to the transformation-based cloaks. By including the PEC coating as the inner layer, the performance is also independent of the material parameters inside of the core.

In conclusion, the analytical model for the multilayered cylindrical cloak has been well established. Based on this model, the effects of discretization and simplification, as well as losses, of the transformation-based cloak can be quantitatively characterized. By utilizing the genetic algorithms, we further show that, it is not the best way to manually assign the transformation parameters for the multilayered cloak. A four-layer quasiperfect cylindrical cloak is proposed, whose parameters are relatively small, and possible for realization. The parameters obtained do not follow the method of coordinate transformation. Our method is shown to be effective in analyzing a multilayered cloak and provides a robust way of designing a practical cloak.

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